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Historical, Canonical, Mathematical and Astronomical Aspects of the Paschalion Question*

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Abstract

There are different calendar systems in use among Orthodox Churches worldwide. Non-movable Orthodox Christian feasts like Nativity, Annunciation, Transfiguration, and so on, are celebrated according to two different - Julian and Revised Julian - calendars. However, when it comes to the question of the Easter date, most of the churches with some exception celebrate the feast of the Resurrection

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of our Lord Jesus Christ on the same Sunday. Despite different calendar systems, it is precisely the Easter date determination question on which all calendar systems are based. Presented paper studies the Easter date (also known as *Paschal*) related question from historical, canonical, mathematical and astronomical points of view. Two existing paschal systems - Alexandrian and Gregorian - are presented here. According to these systems, the dates of the Easter for the period of 2000 - 2050 are calculated and compared with the astronomical dates defined by the Church canonical requirements for the Easter date determination. Obtained results reveal that the Alexandrian method used in most Orthodox churches often deviate from the astronomical reality and canonical rules, and its accuracy can reach only 31% for the given period of years. On the other hand, the accuracy of the Gregorian method used in the Catholic/Protestant world can be as high as 92%.

Keywords

Calendar, Easter, Paschal tables, Alexandrian method, Gregorian Method

1 Introduction

In connection with the Nativity feast, the Church calendar becomes a more and more discussed topic among the believers. However, it is ceased to be such during the Easter period. In fact, the calendar question is directly connected to the Easter date determination: it is precisely the latter that created the calendar issue within the Church. While almost all of us know when the Nativity feast is celebrated by different Orthodox

Christian churches across the world, not many of us can explain why quite often the vast majority of the Orthodox churches celebrate the holiest feast of the feasts – the Resurrection of our Lord Jesus Christ, the Easter – on a Sunday different from the Sunday celebrated by the Catholic and Protestant churches. With this regard year, 2016 was not different: the difference between the Easter celebrations by the Orthodox Christian (except the Orthodox Christian church in Finland, which is under the jurisdiction of the Ecumenical Patriarchate of Constantinople) and the Catholic and Protestant world was five weeks. Namely, we Orthodox Christians celebrated Easter on Sunday, May 1st, while the rest of the Christian world celebrated it on Sunday, March 27th (both dates are according to the Gregorian Calendar). According to the Easter date determination-rule, this feast must be celebrated on the first Sunday following the Paschal full moon, which is the first full moon following or coinciding the vernal equinox. Here, the name "paschal" is derived from "Pascha", which is related to the Aramaic word meaning Passover. Since the Easter formula contains the observable astronomical phenomena, we can easily and independently calculate the Easter date and scientifically verify it. However, we get an entirely different picture for the year of 2016: according to the astronomical data, the vernal equinox took place on March 20th, the first full moon after it was on March 23rd and the following first Sunday was March 27th. This date was exactly the date when Catholic and Protestant churches celebrated the Easter in 2016. That leads to the simple question: how can it be explained that in some years, the Orthodox Easter date coincides with the Catholic/Protestant dates and in other years, the difference between dates can be from one to five weeks with us always celebrating after the Catholic/Protestant world?

The presented article is an attempt to familiarize readers with the issue of Paschal date determination, its roots and reasons, and its implication and results nowadays. During the analysis of this question, the paschal tables, which represent tables with

predetermined Easter dates for the given span of years, will be introduced and their establishment, development, and improvement will be discussed. The algorithms for the Easter date calculation will be explained step by step. In the end, by use of the paschal table technique the Orthodox and Catholic/Protestant Easter dates will be calculated for the period of 2000 – 2050 years and obtained data will be compared with official Paschal dates determined by the Orthodox and Catholic/Protestant churches and also with their corresponding astronomical data.

This paper consists of three chapters: the first chapter deals with the historical and canonical aspects of the Paschal date determination; the second - focuses on the mathematical and astronomical side of this question with detailed analysis, and conclusions based on the obtained data analysis will be given in the final - the third chapter.

2 The Problem of the Paschalion

The Easter is celebrated on the first Sunday, after the first full Moon following or coinciding the vernal equinox[1]. Here, the historical development of this Paschal date formula will be considered.

All four evangelists give us a detailed account of the Resurrection of our Lord Jesus Christ in the New Testament: the Resurrection took place on Sunday after the Jewish feast of

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1 E. G. Richards, *Mapping Time: The Calendar and its History* (Oxford: Oxford University Press, 1999), Ch. 15, p. 599; Archbishop Peter (L'Huillier), The Date of Pascha (The Orthodox Church Newspaper, April-May 1994); Проф. Д. П. Огицкий, Канонические нормы православной пасхалии и проблема датировки Пасхи в условиях нашего времени (Богословские Труды, 7, 1971), p. 207; А. Климишин, *Календарь и хронология* (Москва "Наука", главная редакция, Физико-математической литературы, 1990), p. 93.

Passover had already begun. According to the Old Testament, the Passover is celebrated in Nisan 15th by the Jewish calendar (*Nisan is the first month of the year in the Hebrew calendar and the seventh month (eighth, in a leap year) of the civil year*). As a rule, the Nisan 15th starts with the night of full moon, which follows the vernal equinox. It is known from the history that early Christians before the I Ecumenical Council celebrated the Easter on different days. For example, Christians in Asia Minor and Palestine celebrated this feast during the night from the 14th to the 15th of Nisan, i.e. when the Jews celebrated the Passover. Notable is that the day of the week was not considered, hence it could be any day of the week and not necessarily Sunday. In history, these Christians are known as the Quartodecimans (after the 14th day in Moon calendar)[2]. Alexandrian Christians celebrated the Easter on Sunday between Nisan 15th – 21st. Therefore they had to determine the spring full moon and the following Sunday. Christians in Syria used a similar technique, but their point of origin was different from the one utilized by the Alexandrian school. This resulted in a significant difference in the determination of the Easter dates by the Alexandrian and Syrian schools, which in some cases could reach 4-5 weeks. Contrary to the Alexandrian technique, Syrian calculation was strictly connected to the Jewish calculation, which in some years resulted in Easter dates before the vernal equinox. On the other hand, the bishops in Rome did not allow to celebrate the Easter feast after April 21st, the festive day of the Foundation of Rome ("Parilia" or "Palilia" feast)[3]. By doing so, the bishops did not want the Passion Week to coincide with the Roman festive day. Roman Christians were determining the spring full moon too, but the accuracy of their method was different from the Alexandrian one. Finally, there

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2 Протоиерей В. Ф. Хулап, Реформа календаря и пасхалии: история и современность (Церковный вестник, СПб, No. 3, 2002).

3 А. Климишин, *Календарь и хронология* (Москва "Наука", главная редакция, Физико-математической литературы, 1990), p. 293.

was a different approach to this question among early Christians too: some believed that Crucifixion of Jesus Christ occurred on March 23rd, Friday. Hence, the Sunday, March 25th should have been the date of His Resurrection. However, the March 25th is the date when the Church celebrates the Annunciation Feast, and by that time it was a certain belief that the Universe was created exactly on March 25th. That is why the Easter, which is celebrated in some years on Sunday, March 25th, is called as Kirio Pascha (Note, that this date as "historical" date of our Lord's resurrection was used by Dionysius Exiguus (V-VI AD) while constructing our chronology). Concerning this, there was a Christian sect, which celebrated the Easter exactly on March 25th disregard of which day of the week it was[4].

2.1 Paschal Tables

As the Easter date determination was based on such astronomical phenomena as the vernal equinox, full moon and following Sunday, it was necessary to work out the method, which would give the possibility to calculate in advance the date of Easter. That was exactly how the first paschal tables appeared in early centuries. The Roman and Alexandrian schools independently from each other started creating their tables, which were based on the astronomical data of their time. To achieve highest possible accuracy, it was necessary to bring the solar and lunar calendars as close to each other as possible. However, it is well known from astronomy that the duration of the solar and lunar years are not multiple of each other's i.e. they cannot be divided on each other without a remainder. To solve this problem people already in early history used so-called 8-year and 19-year cycles.

The 8-year cycle, which is older than the 19-year cycle, is based on the observation that the number of days in 8 solar years is

4 Ibid., p. 294.

approximately equal to the number of days in 99 lunar months. It is a shift in the lunar phases constitute ca 1.53 days in 8 years, which is not a high accuracy.
On the other hand, according to the 19-year cycle, which was invented by famous Greek astronomer Meton in 432 BC, the number of the days in 19 solar years was very close to the number of days in 235 lunar months. Indeed, if the average duration of the year according to the Julian calendar is 365.25 days and the average length of the lunar month is 29.530588 days, then:
365.25 x 19 = 6939.75 days
29.530588 x 235 = 6939.68818 days

These two numbers are very close to each other, but they are not equal: time duration of 235 lunar months are shorter than 19 solar years (according to the Julian calendar) by ~1.5 hours. It may look slight discrepancy, but it is not difficult to find out that ~1.5 hours will turn into one day in ~ 305 years, which can no longer be ignored.
Starting from V AD, the 19-year cycle was used to calculate the dates of full moon, which were necessary for the calculations of Easter dates. This method is still in use to determine the paschal full moon nowadays[5]. It must be noted that the Julian calendar itself mentioned above has a built-in inaccuracy compared with a Tropical year. The later represents the time needed for the Sun to return to the same position viewing from the cycle of seasons. In other words, it is the time from e.g. vernal equinox to vernal equinox, or from summer solstice to summer solstice. The duration of the Tropical year is ~365.2422 days, while the length of a year defined by the Julian calendar is 365.25 days. Hence, inaccuracy of the Julian

5 Ibid., p. 131.

Calendar year compared with the Tropical year is one day in ~128 years[6]: 1/(365.25 – 365.2422) ~ 128 years

Concerning the Roman and Alexandrian schools, Alexandrian Christians based their tables on the 19-years Metonic cycle, while the Romans first used the 8-years cycle, and later they changed it to the 84-years cycle (yet another approximation between the solar and lunar years).

As mentioned above, early Christians celebrated the Easter on different from each other dates. Hence, while some Christian groups were celebrating the feast of the Resurrection of our Lord, other groups were still fasting and preparing for the celebration. Also, some Christian groups were "following" the Jewish calculations, which underwent important changes after the destruction of the second temple and extensive depopulation of Judean communities as the result of Bar Kokhba revolt crashing (132-135 AD). Due to those changes, which took place in the period of II – IV AD, the Passover was celebrated in some years before the vernal equinox[7]. This resulted in a very strange picture: in the certain year these Christian groups were celebrating the Easter after the vernal equinox, and in following year – before the vernal equinox. In this time-frame, where the vernal equinox served as starting point for a year, Christians would celebrate the Easter twice within a year. It was evident that different and, in some cases, very radical traditions about the Easter date, were causing disputes between the Christians – a situation which was not acceptable to the Church.

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6 Протоиерей В. Ф. Хулап, Реформа календаря и пасхалии: история и современность (Церковный вестник, СПб, No. 3, 2002).

7 Ibid.

2.3 First Ecumenical Council

In 313 AD St Constantine the Great, the Emperor of Rome proclaimed Edict of Milan, which decreed freedom of religion and tolerance for Christianity in the empire bringing to an end the persecution of Christians. Christianity was slowly acquiring the status of the state religion. For the unity of the empire, a united and vigorous Church was necessary, and when the Trinitarian/Christological disputes between presbyter Arius and St Alexander bishop of Alexandria took place in the Church of Alexandria, which could cause the major division within the Church, the Emperor of Rome upon the recommendations of a synod led by Hosius of Corduba in the Eastertime convened The First Council of Nicaea in 325 AD The Council rejected the Arian teaching and constructed the first part of the Nicaean Creed, promulgated different canon laws and considered the question concerning the observance of the date of Easter.

Unfortunately, no proceedings of the Council on the Easter date have been preserved. They were not in the archives of the Church of Constantinople either. The only preserved document on this issue is the Emperor's letter to those bishops who were not able to attend the Council. The letter stated that the Council "did not find it right to celebrate the Easter according to Jewish way", because according to Jewish calculations "they celebrate Passover twice in certain years"[8]. In other words, it meant not to use the Jewish calculations, which led some Christian groups to celebrate the Easter after vernal equinox in one year and before next vernal equinox in the following year. Since the vernal equinox was used as a kind of division between the years, then one could run into anomaly – celebration of the Easter twice in one year[9]. Exactly in this context, we should

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8 А. Климишин, *Календарь и хронология* (Москва "Наука", главная редакция, Физико-математической литературы, 1990), p. 298.

9 Ibid.; Протоиерей В. Ф. Хулап, Реформа календаря и пасхалии: история и современность (Церковный вестник, СПб, No. 3, 2002).

understand the Apostolic Canon 7, which states: "If any bishop, presbyter, or deacon, shall celebrate the holy day of Easter before the vernal equinox, with the Jews, let him be deposed"[10]. The same can be said about Canon I of the Council of Antioch in 341 AD. These canons are directed not against the celebration of the Easter on the same day on which Jews celebrate Passover, but against following the Jewish calculations; in other words, the canons require to be independent of the Jewish calculations on the question of the determination of the Easter date. However, it is often mentioned nowadays that canons prohibit Christians from celebrating Easter on the same Sunday on which Jews celebrate Passover. This is a misinterpretation of the canons, which derives from wrong comments by the prominent Church Canon Law experts such as Ioan Zonaras (12th-century Byzantine chronicler and theologian), Theodore Balsamon (12th-century Eastern Orthodox Patriarch of Antioch), Matthew Blastares (14th-century Byzantine Greek monk, writer of the Syntagma Canonum). Unfortunately, they more than others contributed to the popularization of this misunderstanding within the Orthodox world[11]. Just to confirm

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10 *The Apostolic Canon 7*: "If any bishop, presbyter, or deacon, shall celebrate the holy day of Easter before the vernal equinox, with the Jews, let him be deposed".

11 Archbishop Peter (L'Huillier), The Date of Pascha (The Orthodox Church Newspaper, April-May 1994); Проф. Д. П. Огицкий, Канонические нормы православной пасхалии и проблема датировки Пасхи в условиях нашего времени (Богословские Труды, 7 (с. 204-211), 1971), p. 205; А. Климишин, *Календарь и хронология* (Москва "Наука", главная редакция, Физико-математической литературы, 1990), p. 299; Протоиерей В. Ф. Хулап, Реформа календаря и пасхалии: история и современность (Церковный вестник, СПб, No. 3, 2002); From the chapter, "The Council of Nicea," in The Church of the Ancient Councils: The Disciplinary Work of the First Four Ecumenical Councils Concerning the Date of Pascha (St. Vladimir's Seminary Press, Crestwood, NY, (pp. 19-26.), 1996); Проф. Иеромонах Л. Воронов, Календарная проблема. Ее изучение в свете решения Первого Вселенского

this statement, it is enough to check the history of Church, according to which starting from III to VIII AD there were numerous cases of coincidences between the Christian Easter and the Jewish Passover dates, but the dates of Easter were never changed due to these coincidences[12]. To summarize, the I Ecumenical Council of Nicea required from Christians to be freed from the dependence on the Jewish calculations, but accidental coincidences between the Easter and Passover dates were not considered as the problem or violation of Church canons.

The Emperor's letter to the bishops also states that all Christians must celebrate the Easter on the same day. However, it was not as easy to fulfill this requirement in the history. Even after the Council of Nicea, there were still two different traditions of the Easter date calculations (known as *paschalion*) – one from Roman and another from Alexandrian school. These various paschallions existed for almost 500 years after the I Ecumenical Council and everything ended by the victory of the Alexandrian school. It should be noted, that the Emperor's letter did not contain any information on the technique for the Easter date determination[13].

Concerning the paschal tables, their development was historically a long process. Most probably it ended between IV – VI AD. They were based on the 19-years Metonic cycle, the

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Собора о пасхалии и изыскание пути к сотрудничеству между Церквами в этом вопросе (Богословские Труды 7, 1971), pp. 170-203, here: p. 189.

12 Проф. Д. П. Огицкий, Канонические нормы православной пасхалии и проблема датировки Пасхи в условиях нашего времени (Богословские Труды, 7 (с. 204-211), 1971), p. 206; А. Климишин, *Календарь и хронология* (Москва "Наука", главная редакция, Физико-математической литературы, 1990), p. 299; Протоиерей В. Ф. Хулап, Реформа календаря и пасхалии: история и современность (Церковный вестник, СПб, No. 3, 2002).

13 Красильников Ю.Д., Солнце, Луна, древние праздники и новомодные теории, (Михаил Городецкий., *Астрономия против "новой хронологии"*, М., Русская панорама, 2001).

method which still is in use nowadays[14]. According to it, first the paschal full moon – the first full Moon after the vernal equinox – is calculated, then the following Sunday is found. It is unknown who was an author of the paschal tables and when exactly they were created. It could be that this method was not officially canonized by the Church. However, it historically became the de-facto standard[15]. Moreover, it is still unknown when the rule requiring the Easter to be celebrated exactly after the vernal equinox was formed[16].

History preserved many unanswered questions around this issue. However, based on what was known later in history and what can be found in the writings of the Church Canon Law experts it is possible to construct the canonical requirements for the Easter date determination [17]:

The Easter must be celebrated:

- On the first Sunday
- After the first full moon, which
- coincides or follows the vernal equinox.

It was agreed that the date of the vernal equinox was fixed on March 21st, the date when the vernal equinox was observed during the I Ecumenical Council in Nicea.

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14 А. Климишин, *Календарь и хронология* (Москва "Наука", главная редакция, Физико-математической литературы, 1990), p. 131.

15 Красильников Ю.Д., *Астрономия против "новой хронологии"*, Солнце, Луна, древние праздники и новомодные теории (М., Русская панорама, 2001).

16 А. Климишин, *Календарь и хронология* (Москва "Наука", главная редакция, Физико-математической литературы, 1990), p. 299.

17 Проф. Д. П. Огицкий, Канонические нормы православной пасхалии и проблема датировки Пасхи в условиях нашего времени (Богословские Труды, 7 (с. 204-211), 1971), p. 207; А. Климишин, *Календарь и хронология* (Москва "Наука", главная редакция, Физико-математической литературы, 1990), p. 93; Archbishop Peter (L'Huillier), The Date of Pascha (The Orthodox Church Newspaper, April-May 1994).

From these data, it is easy to calculate the lower (earliest) and upper (latest) limits for the Easter date. Namely, if the full moon is observed on March 21st and this day is Saturday, then the Easter will be on Sunday, March 22nd. This is the earliest date for the Easter. If the full moon falls on March 19th (according to the "lunar phases tables" it is impossible for the full moon to be observed on March 20), then the Paschal full moon should be the next full moon, which will be on April 18th. If April 18th is Sunday, then the Easter will be celebrated on the following Sunday, which is April 25th. This is the latest date for the Easter. Hence, the lower and upper limits for the Easter celebration are March 22nd and April 25th, correspondingly (both dates are included).

Note the following sensitive data:

- The date of the vernal equinox is fixed on March 21st according to the Julian calendar;
- Inaccuracy of the Julian calendar is one day in ~128 years;
- The inaccuracy of the 19-years Metonic cycle is one day in ~ 305 years.

The Paschal tables were created in IV-VI AD, but because of the mentioned uncertainties in the Julian calendar and the 19-years Metonic cycle, the dates calculated by the tables shifted with time about their corresponding astronomical dates. The method, which was functioning well in IV-VI AD, gradually began to give imprecise results as time passed. For example, the vernal equinox in XVI AD and defined by the Julian calendar was observed on March 10-11 instead of March 21st. At the same time, the paschal full moon determined by the 19-years Metonic cycle for the same century was 3-4 days behind its real astronomical date[18].

It was evident that to fulfill the canonical requirements for the Easter date determination it was necessary to correct those

[18] Красильников Ю.Д., *Астрономия против "новой хронологии"*, Солнце, Луна, древние праздники и новомодные теории (М., Русская панорама, 2001).

uncertainties - it was a need to conduct the Paschal and calendar reform. Exactly this Reformation was initiated by the Roman Catholic Church.

2.4 Pope Gregory XIII Calendar and Paschal Reformation

In 1582 AD Pope Gregory XIII conducted the paschal reformation (papal bull "Inter Gravissimas"[19]). This reformation was based on the project by an Italian doctor, astronomer, philosopher and chronologist Aloysius Lilius (also referred to as Luiji Lilio, 1510-1576 AD). The Papal committee aimed at restoring the reality rooted in the Alexandrian paschallion: March 21st as a date of the vernal equinox and real, astronomical paschal full moon. Besides this, their requirements were to make sure that these dates would not shift with time as it was the case of the Julian calendar: by that time the "Julian" vernal equinox – March 21st - was already behind the real, astronomical vernal equinox by several days (see Table 1).

As the result of this reformation, no new calendar or new moon phases cycle system was introduced. On the contrary, the existing Julian calendar and the 19-years Metonic cycle were corrected. The correction was achieved by (the) introducing leap years both for the Julian calendar and for the Metonic cycle. In particular, all leap years in the Julian calendar remained as the leap years except those who were not multiples of 400; For example, 2000 year is leap year by both Julian and Gregorian calendar, while 2100 is the leap year by the Julian calendar, but usual year by the Gregorian calendar - 2100 is not multiple of 400 (*that's why the difference between both calendars will increase from 13 to 14 days in 2100. Hence, those who use the Julian calendar will have to celebrate the Nativity feast on January 8 instead of January 7, as December 25*

19 *Inter Gravissimas*, Issued by Pope Gregory XIII, February 24, 1581/2.

by Julian calendar will be January 8 according to the Gregorian calendar after 2100[20]). By introducing the system of leap years, the Catholic Church significantly improved accuracy about the Tropical year compared to the one given by the Julian calendar; The Gregorian calendar's inaccuracy became one day in 3333 years, while the Julian calendar's inaccuracy is one day in 128 years.

In the same way, the full moon dates shifts about their astronomical full moon dates were corrected. The authors of the Reformation were well aware that inaccuracy of the 19-yaers Metonic cycle was one day in ~ 305 years. This shift was corrected by introducing the moon or "lunar leap" years – during 2500 years the moon must be rectified eight times: 7 times in every 300 years and the 8th time in remaining 400 years. The correction must take place in the "century" years (i.e. in 1800, 2100, 2400…). According to this scheme, one day correction is done in average every 312.5 years, which cancels well the inaccuracy given by the Metonic cycle, i.e. ~ 305 years[21].

Concerning the limits of the Easter dates: March 22nd – April 25th, they were not changed, but they were counted according to the Gregorian calendar.

Hence, the problem - to bring the Paschallion dates closer to their corresponding astronomical dates – was solved elegantly and with high accuracy.

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[20] Протоиерей В. Ф. Хулап, Реформа календаря и пасхалии: история и современность (Церковный вестник, СПб, No. 3, 2002).

[21] А. Климишин, *Календарь и хронология* (Москва "Наука", главная редакция, Физико-математической литературы, 1990), p. 301; Красильников Ю.Д., *Астрономия против "новой хронологии"*, Солнце, Луна, древние праздники и новомодные теории (М., Русская панорама, 2001).

Century	Difference in days	Century	Difference in days
III	0	XIII	7
IV	1	XIV	8
V	1	XV	9
VI	2	XVI	10
VII	3	XVII	10
VIII	4	XVIII	11
IX	4	XIX	12
X	5	XX	13
XI	6	XXI	13
XII	7	XXII	14

Table 1: Difference between the Julian and Gregorian calendars

2.5 Paschal/Calendar Reformation Attempts in the Orthodox World

Existing inaccuracies in the calendar were noticed not only in the Catholic but also in the Orthodox world. Here are few examples:

- In XIV century Byzantine astronomer, historian, and theologian Nicephorus Gregoras and Byzantine mathematician and monk Isaac Argyros pointed out to the Byzantine emperor Andronikos II (1282 – 1328 AD) these inaccuracies and asked to conduct the calendar reform. Unfortunately, the reform did not take place. That was most probably due to the lack of support from political and social circles. There was not even attempt to improve the calendar because it was believed then that year 1492 would mark 7000 years from the Creation of Universe. Hence, some believers were in expectation of the Lord's second coming[22].

[22] Протоиерей В. Ф. Хулап, Реформа календаря и пасхалии: история и современность (Церковный вестник, СПб, No. 3, 2002); R. Guiland, Essai sur Nicephore Gregoras (Paris: P. Geuthner, 1926), pp. 282-284; Dictionnaire de Théogogie Catholique, Tome 11 (Paris,

- The Church Council of Constantinople in 1583 acknowledged the fact that there were inaccuracies in the Julian calendar but refused to accept the Gregorian reform[23].
- There were several attempts to consider a question concerning the calendar reform in Orthodox Russia. The Russian Academy of Science expressed interest in this question in 1830, but one of the princes (K. A. Liven) convinced Emperor Nicholas I that people would not be ready for this kind of changes and that it would lead to unrest. Therefore, the question was never considered. In 1846 appeared I. G. Medler's article with title "Concerning the calendar reform", where he proposed to correct the Julian calendar by removing one day from the calendar every 128 years. However, his initiative did not succeed either[24].
- The Pan-Orthodox Congress of Constantinople in 1923 decided to celebrate the Easter according to the formula - "first Sunday following the first full moon after the vernal equinox" with all parameters defined not according to the Alexandrian paschalion, but according to the astronomical data of Jerusalem meridian. This decision has never been implemented by the Orthodox churches. Besides this, the Congress decided to bring into use the Revised Julian calendar developed and proposed by the Serbian (!) mathematician, astronomer, climatologist, geophysicist, civil engineer, doctor of technology, university professor and popularizer of science Milutin Milanković (1879 – 1958)[25]. When it comes to the astronomical data, the Revised Julian calendar is more precise than the Gregorian one due to the improved scheme of leap years. According to this scheme,

1911), col. 455; M. Welborn, "Calendar Reform in the 13th Century" (Chicago: University of Chicago Dissertation, 1935), p. 31.

23 А. Климишин, *Календарь и хронология* (Москва "Наука", главная редакция, Физико-математической литературы, 1990), p. 306.

24 Ibid.

25 M. Milankovitch, Das Ende des julianischen Kalenders und der neue Kalender der orientalischen Kirchen (Astronomische Nachrichten 220, 1924), pp. 379-384.

the discrepancy between the Revised Julian calendar year and the Tropical year is reduced further with the inaccuracy of one day in *ca* 40000 years[26]. Many, but not all, Orthodox churches gradually adopted the Revised Julian calendar for the celebration of non-movable feasts. It must be noted, that this calendar was twice introduced in the Georgian Autocephalous Orthodox Church in 1920s, but both attempts were unsuccessful, and the Julian calendar was restored at the end[27]. A similar picture was observed in Russia – the Russian Autocephalous Orthodox Church adopted the Revised Julian calendar in 1920s, but for very short time and went back to the Julian calendar. The reason in both cases was the same – parishioners and clergy were not ready for this reform[28].

- In 1948 the Pan-Orthodox Congress took place in Moscow. Among different questions considered by the Congress was the questions of calendar and Paschal. It was decided that all Orthodox churches have to celebrate the Easter according to the Alexandrian paschalion based on the Julian calendar, while non-movable feasts should be celebrated by that calendar, which was in use in a given autocephalous church[29]. It must be noted, that earlier Moscow Patriarchate allowed French and Dutch Orthodox parishioners placed under its jurisdiction to celebrate Easter according to the new style (Gregorian calendar) as the part of the Church oikonomia and which was

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26 Протоиерей В. Ф. Хулап, Реформа календаря и пасхалии: история и современность (Церковный вестник, СПб, No. 3, 2002).

27 *The Georgian Church Calendar* (The Georgian Orthodox Church Holy Synod Press, 1928, pp. 33-39); Priest I. Chigladze, The Georgian Orthodox Church and so called "new style" (the Revised Julian Calendar) (http://www.orthodoxtheology.ge/eklesia-kalendari/).

28 Протоиерей В. Ф. Хулап, Реформа календаря и пасхалии: история и современность (Церковный вестник, СПб, No. 3, 2002).

29 Деяния совещания глав и представителей автокефальных православных церквей в связи с празднованием 500-летия автокефалии русской православной церкви, 1948 год (http://krotov.info/history/20/1940/1948_00.htm).

later even more extended. Namely, according to the decision of the Holy Synod of the Moscow Patriarchate in 1967:

> "From the experience of the early church, when the East and the West (bishops in Rome and Asia) celebrated Easter on different days, but their community and prayer unity was preserved, and from the experience of the Finnish Orthodox Church and our parishes in Nederland and, especially, from the experience of the Lord's Resurrection parish surrounded by other religious environment, it must be allowed to our Orthodox parishioners resided in Switzerland and being under the jurisdiction of the Moscow Patriarchate to celebrate non-movable feasts and the feasts of the paschal circle according to the new style"[30].

Moreover, this practice still continues nowadays[31].

2.6 World Council of Churches' Proposal in 1997

In 1997 Orthodox, Catholic and Protestant delegates during the meeting in Aleppo, Syria, adopted a recommendation to all members of the World Council of Churches to work out common method for the Easter date determination, and that this approach instead of using calculation should be based on three basic principles of Nicea and astronomical observation made on Jerusalem Meridian[32]. This recommendation is similar

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[30] Журнал Московской Патриархии (№ 8. С. 1, 1967).

[31] The Monastery of the icon of Holy Mother of God "Joy of All Who Sorrow" in Pervijze, Belgium (Russian Orthodox Church in Nederlands and Belgium, Moscow Patriarchate) is using the new style for the celebration of non-movable and movable feasts (http://archiepiskopia.be/index.php?content=article&category=parishes/be&id=14_perwijze&lang=ru).

[32] World Council of Churches / Middle East Council of Churches Consultation, "Towards a Common Date for Easter" (1997): http://www.oikoumene.org/en/resources/documents/wcc-commissions/faith-and-order-commission/i-unity-the-church-and-its-mission/towards-a-common-date-for-easter/index?set_language=en.

to the decision mentioned above, made by the Pan-Orthodox Congress of Constantinople in 1923. However, it has not been realized in practice.

To conclude, we have the following picture concerning the church calendar within the Orthodox world:

Orthodox churches (excluding the Finnish Autonomous Orthodox Church, which is under the jurisdiction of Ecumenical Patriarchate of Constantinople; and at least one parish in Western Europe, which is under the jurisdiction of Moscow Patriarchate) celebrate Easter according to the Alexandrian paschalion using the Julian calendar.

When it comes to the non-movable feasts, Orthodox autocephalous churches, such as Ecumenical Patriarchate of Constantinople, Patriarchate of Alexandria, Patriarchate of Antioch, Patriarchate of Romania, Patriarchate of Bulgaria, Church of Greece, Church of Cyprus, Church of Albania, Church of the Czech Lands and Slovakia, Orthodox Church in America, and partially Church of Poland, follow the Revised Julian calendar, while Patriarchate of Jerusalem, Patriarchate of Georgia, Patriarchate of Moscow, Patriarchate of Serbia, partially Church of Poland, follow the Julian calendar. Note, that the monasteries on the Mount Athos use the Julian calendar, while they are under the jurisdiction of the Ecumenical Patriarchate of Constantinople, which follows the Revised Julian calendar.

3 Paschal Tables

The calculation of the Paschal tables and their method is complex, but not difficult. Without going into a detailed

explanation, which can be found throughout the literature[33], the usage of these methods will be considered here.
First of all, a few words on the terminology used here:
1) For the simplicity reasons, the "Julian Calendar" will be replaced by the "old style."
2) "The Gregorian" and "Revised Julian Calendar" (because they give the same results until 2800) will be replaced by the "new style."
3) Mathematical operation of MODULO with shorter form of "MOD" will be used here. It finds a remainder after the division of one number by another. For example:
The remainder of 10 divided by 8 is MOD[10/8] = **2**;
i.e. (10 = 1*8 + **2**)
The remainder of 16 divide by 8 is: MOD[16/8] = **0**;
i.e. (16 = 2*8 + **0**)
The remainder of 2016 divided by 19 is: MOD[2016/19] = **2**
i.e. (2016 = 106*19 + **2)**
4) Mathematical operation of rounding down, which rounds the number to the nearest lower integer. Its expression is "INT". For example:
INT[5.3] = 5
INT[19.1] = 19
INT[30.9] = 30

3.1 Table Method - Alexandrian Paschalion

To find the Easter date for a given year according to the Alexandrian paschalion, the year must be divided by 19 (due to the 19-year Metonic Cycle), and its remainder must be found:
N = MOD [Year/19].

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[33] А. Климишин, *Календарь и хронология* (Москва "Наука", главная редакция, Физико-математической литературы, 1990), p. 131; Ж. Меес, *Астрономические формулы для калькуляторов* (М., "Мир", 1988), p. 26-33; Reinhold Bien, *Gauss and Beyond: The making of Easter Algorithms* (Arch. Hist. Exact Sci. 58, 2004), pp. 439-452.

Once the remainder is calculated and by using the Table 2 the paschal full moon (PFM in short) date can be found for that year. Note, that the PFM represents the predefined date of the full moon (and not the actual astronomical full moon, which, in principle, it was meant to be). Since the obtained date is according to the old style, to convert it to the new style 13 days must be added if conversion is done for the XX-XXI centuries. Afterward, the first Sunday following converted PFM date (according to the new style) will be the Easter Sunday according to the new style. If, however, the PFM falls on Sunday, then the following Sunday will be the Easter Sunday[34].

N	Paschal full moon (old style)	N	Paschal full moon (old style)
0	April 5	9	March 27
1	March 25	10	April 15
2	April 13	11	April 4
3	April 2	12	March 24
4	March 22	13	April 12
5	April 10	14	April 1
6	March 30	15	March 21
7	April 18	16	April 9
8	April 7	17	March 29
		18	April 17

Table 2. Paschal full moon dates according to the Alexandrian Paschalion

[34] А. Климишин, *Календарь и хронология* (Москва "Наука", главная редакция, Физико-математической литературы, 1990), p. 130; Reinhold Bien, *Gauss and Beyond: The making of Easter Algorithms* (Arch. Hist. Exact Sci. 58, 2004), pp. 439-452.

For example, let's calculate the Easter dates for the years 2016, 2017 and 2018.
Data for the year 2016:
MOD[2016/19] = 2.
According to the Table 2, the obtained remainder of 2 gives April 13 (old style) for the PFM date. Conversion to the new style gives April 26. Since the date for the PFM is found, by using any standard calendar one can get a day of a week for the calculated PFM, and its following Sunday will be the Easter Sunday. Hence, the Paschal full moon in 2016 was on Tuesday, April 26 by the new style, the Easter in 2016 was on Sunday, May 1st by new style.
Data for the year 2017:
MOD[2017/19] = 3.
According to the Table 2, the obtained remainder of 3 gives April 2 (old style) for the PFM date. Hence, the Paschal full moon in 2017 will be on Saturday, April 15 by the new style, the Easter in 2017 will be on Sunday, April 16 by new style.
Data for the year 2018:
MOD[2018/19] = 4.
According to the Table 2, the obtained remainder of 4 gives March 22nd (old style) for the PFM date. Hence, the Paschal full moon in 2018 will be on Wednesday, April 4th by the new style, the Easter in 2018 will be on Sunday, April 8th by new style.
Obtained results are in full agreement with the official Easter dates found in any Orthodox Church calendar[35].

3.2 Table Method - Gregorian Reform

Similarly to the Alexandrian paschalion, according to the Gregorian paschal reform first the Paschal full moon must be determined and then the first following Sunday will be the

[35] For the information on the Orthodox Paschal dates for the period of 1800-2100 refer to following site: http://www.orthodoxchristian.info/pages/Pascha_dates.htm.

Easter Sunday. To do so, in the beginning, an Epact of the given year is calculated, where the Epact is the age of the moon in days (i.e. from 0 to 29) on January 1. Afterward, the PFM is defined from the special table (see. Table 3), which describes the relation between the Epacts and the PFM. The Epact formula takes into account corrections for the inaccuracies of the Julian calendar ("solar" equitation) and of the moon phases of the 19-years Metonic cycle ("lunar" equation)[36]:

Here are the steps for the Epact calculation:

For the given year later than 1582 a "Golden Number", gn is calculated:

gn = MOD[Year/19] + 1.

Then, the following is found:

- The Restored Epact, epr: epr = MOD[((11*gn) - 10)/30];
- The Century factor, cy: cy = INT[Year/100] + 1;
- The "solar" equation, sol: sol = INT[(3*cy)/4] – 12;
- The "lunar" equation, lun: lun = INT[(8*cy + 5)/25] – 5;
- The Principal Epact, epp: epp = MOD[(-sol+lun)/30]
- Finally, the Gregorian Epact, epg: epg = MOD[(epr-sol+lun)/30] = MOD[(epr+epp)/30]

Here, if epg = 24, and/or epg = 25 and gn > 11, then (epg = epg +1).

Epact	Paschal full moon (new style)	Epact	Paschal full moon (new style)	Epact	Paschal full moon (new style)
0	April 13	10	April 3	20	March 24
1	April 12	11	April 2	21	March 23
2	April 11	12	April 1	22	March 22
3	April 10	13	March 31	23	March 21
4	April 9	14	March 30	24	April 18
5	April 8	15	March 29	25	April 18
6	April 7	16	March 28	25*	April 17
7	April 6	17	March 27	26	April 17
8	April 5	18	March 26	27	April 16
9	April 4	19	March 25	28	April 15
				29	April 14

[36] Reinhold Bien, *Gauss and Beyond: The making of Easter Algorithms* (Arch. Hist. Exact Sci. 58, 2004), pp. 439-452.

Table 3. Relation between the paschal full moon and Epacts according to the Gregorian reform

The following decisions by the papal calendar commission are included in these calculations[37]:
1) In the case of the Epact 24, it is possible in some years that the Easter Sunday falls on April 26th, which is beyond the Easter date limits (just to recall, the limits are March 22nd – April 25th). Therefore, the Epact 24 will be replaced by the Epact 25 and the Easter will be moved to previous Sunday, which will be April 19.
2) During the same 19-year cycle of the Golden Number, the paschal full moon must not fall on the same date twice. To ensure this requirement, the additional Epact 25*, similar to Epact 26, is introduced. In other words, in the case of the Epact 25 and if 12<gn<19, or if both Epacts 24 and 25 are present within the same 19-year cycle, then the Epact 25 will be replaced by the Epact 25*. Likewise, in the case of the Epact 25 and if 1<gn<11, then usual Epact 25 is used.
As an example, the calculation of the Easter dates for the years of 2016, 2017 and 2018 are given below (all dates are according to the new style):
Data for year 2016:
gn = 3; epr = 23; cy = 21; sol = 3; lun = 1; epp = 28;
Epact: epg = 21
PFM: March 23rd, Wednesday.
Easter: March 27th, Sunday.
Data for the year 2017:
gn = 4; epr = 4; cy = 21; sol = 3; lun = 1; epp = 28;
Epact: epg = 2.
PFM: April 11th, Tuesday.
Easter: April 16th, Sunday.
Data for the year 2018:

37 Ibid., pp. 439-452.

gn = 5; epr = 15; cy = 21; sol = 3; lun = 1; epp = 28;
Epact: epg = 13.
PFM: March 31st, Saturday.
Easter: April 1st, Sunday.
Obtained results are in full agreement with the official Easter dates determined in the Catholic/Protestant world[38].

3.3 Analysis of the obtained results through the prism of the astronomical data

The Easter and the Paschal full moon dates calculated for (the in) 2016 according to the Alexandrian and Gregorian methods differ significantly from each other. This does not happen only for 2016. To understand these differences, obtained data must be compared with their corresponding astronomical data taking into account all inaccuracies due to the calendars. Here, the astronomical data means the real, astronomically observed data, which corresponds to the church-canonical requirements for the Easter date determination. Namely, the first real, astronomical full moon, which coincides or follows 21st of March, and the first Sunday after this full moon.
Let's recall once again the criteria for the Easter date determination and investigate which paschalion/calendar system gives more precise and accurate results:

The Easter date formula
The Easter must be celebrated on
- the first Sunday, which follows;
- the first full moon, which coincides or follows;
- the vernal equinox.

[38] For the information on the Catholic Pachal dates refer to the following site: https://www.calendar-12.com/catholic_holidays/.

The date of the vernal equinox
- Historically the date of the vernal equinox was on the 21st of March. According to the Gregorian Reformation, the date of the vernal equinox was fixed on the 21st of March by the new style. The 21st of March was the day when the vernal equinox was observed during the I Ecumenical Council in Nicea. Hence, the date of the vernal equinox is the 21st of March according to the new style.

The limits of the Easter dates
- March 22nd – April 25th including both dates.

Inaccuracies of the calendar
- Inaccuracy of the Julian calendar: 1 day in ~128 years;
Because of this fact, the Vernal Equinox according to the Julian calendar is observed not on the 21st of March, but on earlier dates: in XX-XXI century, the vernal equinox is observed on March 8th by the Julian Calendar (which is 21st of March by the Gregorian Calendar).
- The inaccuracy of the 19-years Metonic cycle: 1 day in ~ 305 years.
Because of this fact, the paschal full moon calculated by the Alexandrian Paschal Tables is more and more behind its corresponding real, astronomical full moon as time progresses.

Paschal/calendar systems
-The Gregorian paschal system is used in the Catholic and Protestant Christian world;
- The Alexandrian Paschal system with the 19-years Metonic cycle and the Julian calendar is used in the Orthodox Christian world (excluding the Finnish Orthodox Church, which uses the new style for the celebration of both movable and non-movable feasts).
Table 4 shows results of the Easter and Paschal full moon dates and their corresponding astronomical data for the period of 2000 – 2050 years calculated by the Gregorian paschalion. All

dates are according to the new style. Note, that the astronomical data – the first full moon on or after the 21st of March and following Sunday – were calculated according to the criteria mentioned above (the astronomical data on the full moon are available on the Internet[39]).

Year	Astronomical full moon	Paschal full moon	First Sunday after astronomical full moon	Easter Sunday
2000	18, April, Tuesday	18, April, Tuesday	April 23	April 23
2001	8, April, Sunday	8, April, Sunday	April 15	April 15
2002	28, March, Thursday	28, March, Thursday	March 31	March 31
2003	16, April, Wednesday	16, April, Wednesday	April 20	April 20
2004	5, April, Monday	5, April, Monday	April 11	April 11
2005	25, March, Friday	25, March, Friday	March 27	March 27
2006	13, April, Thursday	13, April, Thursday	April 16	April 16
2007	2, April, Monday	2, April, Monday	April 8	April 8
2008	21, March, Friday	22, March, Saturday	March 23	March 23
2009	9, April, Thursday	10, April, Friday	April 12	April 12
2010	30, March, Tuesday	30, March, Tuesday	April 4	April 4
2011	18, April, Monday	17, April, Sunday	April 24	April 24
2012	6, April, Friday	7, April, Saturday	April 8	April 8
2013	27, March, Wednesday	27, March, Wednesday	March 31	March 31
2014	15, April, Tuesday	14, April, Monday	April 20	April 20
2015	4, April, Saturday	3, April, Friday	April 5	April 5
2016	23, March, Wednesday	23, March, Wednesday	March 27	March 27
2017	11, April, Tuesday	11, April, Tuesday	April 16	April 16
2018	31, March, Saturday	31, March, Saturday	April 1	April 1
2019	21, March, Thursday / 19, April, Friday	18, April, Thursday	March 24	April 21
2020	8, April, Wednesday	8, April, Wednesday	April 12	April 12
2021	28, March, Sunday	28, March, Sunday	April 4	April 4
2022	16, April, Saturday	16, April, Saturday	April 17	April 17
2023	6, April, Thursday	5, April, Wednesday	April 9	April 9
2024	25, March, Monday	25, March, Monday	March 31	March 31
2025	13, April, Sunday	13, April, Sunday	April 20	April 20
2026	2, April, Thursday	2, April, Thursday	April 5	April 5
2027	22, March, Monday	22, March, Monday	March 28	March 28
2028	9, April, Sunday	10, April, Monday	April 16	April 16
2029	30, March, Friday	30, March, Friday	April 1	April 1
2030	18, April, Thursday	17, April, Wednesday	April 21	April 21
2031	7, April, Monday	7, April, Monday	April 13	April 13
2032	27, March, Saturday	27, March, Saturday	March 28	March 28
2033	14, April, Thursday	14, April, Thursday	April 17	April 17
2034	3, April, Monday	3, April, Monday	April 9	April 9
2035	23, March, Friday	23, March, Friday	March 25	March 25
2036	10, April, Thursday	11, April, Friday	April 13	April 13
2037	31, March, Tuesday	31, March, Tuesday	April 5	April 5
2038	21, March, Sunday / 19, April, Monday	18, April, Sunday	March 28	April 25
2039	9, April, Saturday	8, April, Friday	April 10	April 10
2040	28, March, Wednesday	28, March, Wednesday	April 1	April 1
2041	16, April, Tuesday	16, April, Tuesday	April 21	April 21
2042	5, April, Saturday	5, April, Saturday	April 6	April 6
2043	25, March, Wednesday	25, March, Wednesday	March 29	March 29
2044	12, April, Tuesday	13, April, Wednesday	April 17	April 17
2045	1, April, Saturday	2, April, Sunday	April 2	April 9
2046	22, March, Thursday	22, March, Thursday	March 25	March 25
2047	10, April, Wednesday	10, April, Wednesday	April 14	April 14
2048	30, March, Monday	30, March, Monday	April 5	April 5
2049	18, April, Sunday	17, April, Saturday	April 25	April 18
2050	7, April, Thursday	7, April, Thursday	April 10	April 10

Table 4. The Paschal table data according to the Gregorian Reformation and their corresponding astronomical data. Data marked by the gray color represent the cases when there is a discrepancy between the Paschal and astronomical data.

39 For the information on the full moon dates for the period of 1900 – 2100 refer to the following site: http://home.hiwaay.net/~krcool/Astro/moon/fullmoon.htm.

Obtained results confirm that the Gregorian method finds the paschal full moon with a quite good approximation. However, there are few discrepancies which can be sorted out in two different classes:

1) The difference between the astronomical and Paschal table data with regards of the full moon is (+/-) 1 day; This discrepancy is attributed to the fact that the correction of the moon cycles does not happen dynamically, but only in certain years. This (+/-) 1-day difference in some years gives us (+/-) 1-week difference between the Easter dates determined astronomically and by the Paschal tables. In particular:

- The Paschal full moon in the 2045 year will fall on Sunday, April 2. Hence, the following Sunday, April 9th will be the Easter Sunday. However, the astronomical full moon will fall on Saturday, April 1st. Hence, the next Sunday, April 2nd must be the Easter Sunday.
- The picture for the 2049 year is opposite to the one for 2045. Namely, the paschal full moon according to the paschal tables will be on Saturday, April 17th. Therefore, the following Sunday, April 18th will be the Easter Sunday. However, the astronomical full moon will be observed on Sunday, April 18th. Hence, the next Sunday, April 25th must be the Easter Sunday.

2) According to above-mentioned calculations and by use of the paschal tables, the Gregorian Epacts for the 2019 and 2038 years is 24 (and not 23). Therefore, the paschal full moon must be on April 18th (and not March 21st). However, according to the astronomical data, the full moon in mentioned years will occur in the very early morning of the 21st of March (01:44 GMT in the 2019 year and 02:11 GMT in the 2038 year). Hence, astronomically speaking, the second full moon after the 21st of March will be considered as the paschal full moon by the Paschal tables. That is why the difference between the Paschal and "astronomical" Easter dates are monitored in the mentioned years.

To summarize, the Easter dates determined by the Gregorian method in the given period of years (51 years) coincides with

their corresponding astronomical data 47 times. That gives ca 92% accuracy, which is pretty high value.

Table 5 illustrates the correlation between the Easter dates provided by the Alexandrian paschalion and astronomical data for the period of 2000 – 2050 years. The dates are according to the new style.

Year	Astronomical full moon	Paschal full moon	Full moon after March 21	First Sunday after astronomical full moon	Easter Sunday	Easter Sunday after astronomical full moon
2000	18, April, Tuesday	23, April, Sunday	1	April 23	April 30	2
2001	8, April, Sunday	12, April, Thursday	1	April 15	April 15	1
2002	28, March, Thursday / 27, April, Saturday	1, May, Wednesday	2	March 31	May 5	2
2003	16, April, Wednesday	20, April, Sunday	1	April 20	April 27	2
2004	5, April, Monday	9, April, Friday	1	April 11	April 11	1
2005	25, March, Friday / 24, April, Sunday	28, April, Thursday	2	March 27	May 1	1
2006	13, April, Thursday	17, April, Monday	1	April 16	April 23	2
2007	2, April, Monday	6, April, Friday	1	April 8	April 8	1
2008	21, March, Friday / 20, April, Sunday	25, April, Friday	2	March 23	April 27	1
2009	9, April, Thursday	14, April, Tuesday	1	April 12	April 19	2
2010	30, March, Tuesday	3, April, Saturday	1	April 4	April 4	1
2011	18, April, Monday	22, April, Friday	1	April 24	April 24	1
2012	6, April, Friday	11, April, Wednesday	1	April 8	April 15	2
2013	27, March, Wednesday / 25, April, Thursday	30, April, Tuesday	2	March 31	May 5	2
2014	15, April, Tuesday	18, April, Friday	1	April 20	April 20	1
2015	4, April, Saturday	7, April, Tuesday	1	April 5	April 12	2
2016	23, March, Wednesday / 22, April, Friday	26, April, Tuesday	2	March 27	May 1	2
2017	11, April, Tuesday	15, April, Saturday	1	April 16	April 16	1
2018	31, March, Saturday	4, April, Wednesday	1	April 1	April 8	2
2019	19, April, Friday	23, April, Tuesday	1	April 21	April 28	2
2020	8, April, Wednesday	12, April, Sunday	1	April 12	April 19	2
2021	28, March, Sunday / 27, April, Tuesday	1, May, Saturday	2	April 4	May 2	1
2022	16, April, Saturday	20, April, Wednesday	1	April 17	April 24	2
2023	6, April, Thursday	9, April, Sunday	1	April 9	April 16	2
2024	25, March, Monday / 23, April, Tuesday	28, April, Sunday	2	March 31	May 5	2
2025	13, April, Sunday	17, April, Thursday	1	April 20	April 20	1
2026	2, April, Thursday	6, April, Monday	1	April 5	April 12	2
2027	22, March, Monday / 20, April, Tuesday	25, April, Sunday	2	March 28	May 2	2
2028	9, April, Sunday	14, April, Friday	1	April 16	April 16	1
2029	30, March, Friday	3, April, Tuesday	1	April 1	April 8	2
2030	18, April, Thursday	22, April, Monday	1	April 21	April 28	2
2031	7, April, Monday	11, April, Friday	1	April 13	April 13	1
2032	27, March, Saturday / 25, April, Sunday	30, April, Friday	2	March 28	May 2	1
2033	14, April, Thursday	18, April, Monday	1	April 17	April 24	2
2034	3, April, Monday	7, April, Friday	1	April 9	April 9	1
2035	23, March, Friday / 22, April, Sunday	26, April, Thursday	2	March 25	April 29	1
2036	10, April, Thursday	15, April, Tuesday	1	April 13	April 20	2
2037	31, March, Tuesday	4, April, Saturday	1	April 5	April 5	1
2038	19, April, Monday	23, April, Friday	1	April 25	April 25	1
2039	9, April, Saturday	12, April, Tuesday	1	April 10	April 17	2
2040	28, March, Wednesday / 27, April, Friday	1, May, Tuesday	2	April 1	May 6	2
2041	16, April, Tuesday	20, April, Saturday	1	April 21	April 21	1
2042	5, April, Saturday	9, April, Wednesday	1	April 6	April 13	2
2043	25, March, Wednesday / 24, April, Friday	28, April, Tuesday	2	March 29	May 3	2
2044	12, April, Tuesday	17, April, Sunday	1	April 17	April 24	2
2045	1, April, Saturday	6, April, Thursday	1	April 2	April 9	2
2046	22, March, Thursday / 20, April, Friday	25, April, Wednesday	2	March 25	April 29	2
2047	10, April, Wednesday	14, April, Sunday	1	April 14	April 21	2
2048	30, March, Monday	3, April, Friday	1	April 5	April 5	1
2049	18, April, Sunday	22, April, Thursday	1	April 25	April 25	1
2050	7, April, Thursday	11, April, Monday	1	April 10	April 17	2

Table 5. The Easter and the Paschal full moon dates according to the Alexandrian Paschal tables and their corresponding astronomical data. Here, the column "Astronomical full moon" shows the dates of the full moons taking place before the paschal full moon determined by the Alexandrian Paschal tables. Their amount is given in the column "Full moon after March 21". Similarly, the last column "The Easter Sunday after astronomical full moon" shows whether the Easter Sunday is the first or second Sunday after the real astronomical full moon. Data marked by the gray color represent the cases when the Paschal and astronomical data do not match each other.

Obtained results clearly show that, contrary to the results by the Gregorian method, the Easter and the Paschal full moon dates determined by the Alexandrian paschalion do not often match to their corresponding astronomical dates. Within the given 51 years of period, they match only 16 times (i.e. in years of 2001, 2004, 2007,2010, 2011, 2014, 2017, 2025, 2028, 2031, 2034, 2037, 2038, 2041, 2048, 2049). This indicates to very low – ca 31% accuracy.

Due to the inaccuracy in the 19-years Metonic cycle used in the Alexandrian paschalion, the paschal full moon is 3-5 days behind the real, astronomical full moon. Moreover, the vernal equinox in the Alexandrian system is defined as March 21st according to the old style (and not new style!). March 21st of the old style is April 3rd of the new style. In other words, according to the Julian calendar (old style) and the Alexandrian system, the vernal equinox will take place 13 days after the March 21st by the new style. March 21st is the date for the vernal equinox by the new style in the Gregorian system (note, however, that the real, astronomical vernal equinox may be different from March 21st by 1-2 days, which will be shown below). Hence, if the actual full moon takes place between March 21st – April 3rd, it will not be considered as the first full moon. The paschal full moon will be the first full moon on or after April 3rd. This is also confirmed by given table data.

For more clarification, two columns are inserted in Table 5:

1) "Full Moon after March 21st" shows how many full moons took place between the vernal equinox (fixed to March 21st by new style) and the Paschal full moon determined by the Alexandrian paschalion.

2) "The Easter Sunday after astronomical full moon" indicates the order (1st or 2nd) of the Alexandrian Easter Sunday after the astronomical full moon.

Diagram 1-3 gives a graphical representation of the obtained results for years of 2016, 2017 and 2018. For example, in 2016 the first full moon after the vernal equinox (March 21st, new style) was on Wednesday, March 23rd. Hence, the Easter Sunday

should have been the first Sunday after the March 23rd, which was March 27th. This was the Easter Sunday in the Catholic/Protestant world. In the Orthodox Christian world, the vernal equinox is on March 21st according to the old style, which corresponds to the April 3rd by new style. Hence, the full moon on March 23rd could not be considered as the first full moon after the vernal equinox. Therefore, the next - the full moon after April 3rd, must be considered. Such astronomical full moon was observed on Friday, April 22nd. However, according to the Paschal tables and discussed above inaccuracies in the system, the next full moon would take place on Tuesday, April 26th (Picture 1 shows the real phases of the moon for mentioned dates). Sunday, May 1st, after this (i.e. April 26th) full moon and not after the real, astronomical (i.e. April 22nd) full moon was the Easter Sunday determined according to the Alexandrian paschalion and celebrated throughout Orthodox Christian world with mentioned exception. As a result, the Easter date in 2016 determined by the Alexandrian paschalion was the 2nd (and not the 1st) Sunday after the astronomical full moon. On the other hand, the paschal full moon, which was already behind the real astronomical full Moon by four days, was the 2nd (and not the 1st) full moon after the vernal equinox. Hence, the canonical requirement for the Easter date was broken twice in 2016: the Easter in 2016 was celebrated on the 2nd Sunday after the 2nd full moon after the vernal equinox.

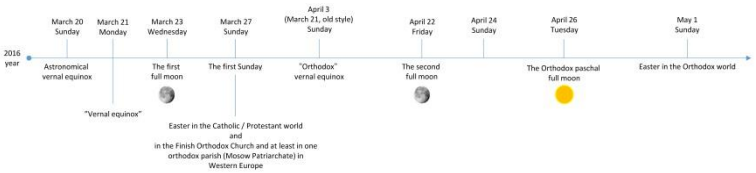
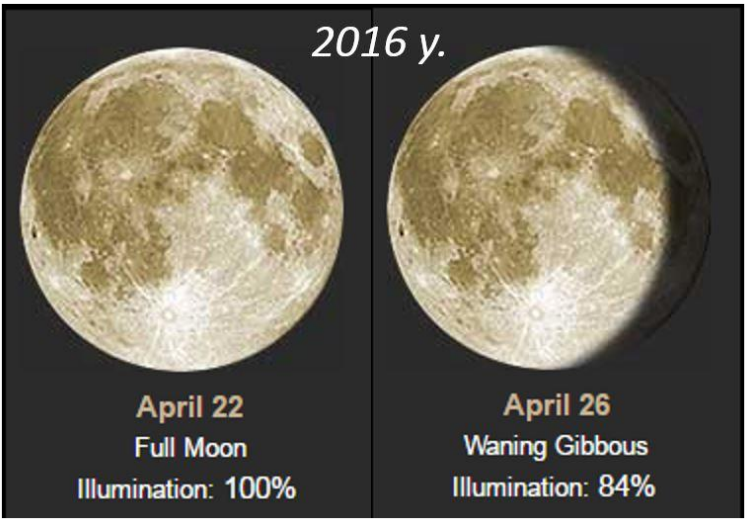


Diagram 1. Graphical representation of the Easter dates by the Alexandrian and Gregorian paschalion together with their corresponding astronomical dates for 2016 y.



Picture 1. 2016 y. Moon phases for the April 22nd - astronomical full moon (on the left) and April 26th – Alexandrian Paschal full moon (on the right)[40]

Contrary to 2016, Easter in 2017 will be celebrated on the same day, Sunday / April 16th, in the Catholic/Protestant and Orthodox Christian world. It is because:

1) The first full moon after the March 21st will be on April 11th, which is after the "Orthodox" vernal equinox (April 3rd by the new style);
2) According to the Alexandrian paschalion, the paschal full moon for 2017 will be on Saturday, April 15th. Hence, Sunday, April 16th will be the Easter Sunday.

[40] For the information on the Moon phases refer to the following site: http://www.moongiant.com/.

Diagram 2. Graphical representation of the Easter dates by the Alexandrian and Gregorian paschalion together with their corresponding astronomical dates for 2017 y.

In 2018, the picture is slightly different: Orthodox Easter Sunday is after the first astronomical full moon after the vernal equinox, but it will be in the second and not on the first Sunday after this full moon (see Diagram 3).

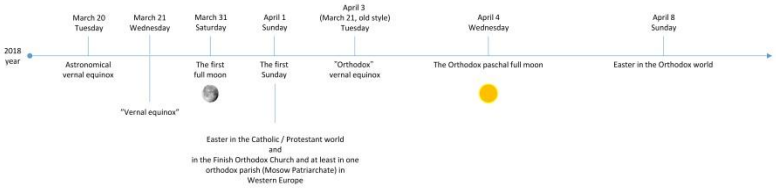


Diagram 3. Graphical representation of the Easter dates by the Alexandrian and Gregorian paschalion together with their corresponding astronomical dates for 2018 y.

The interested reader is invited to self-construct the similar diagrams for other years. For example, the year 2021 can be considered when the Easter Sunday according to the Alexandrian paschalion will be the first Sunday after the second astronomical full moon after the vernal equinox. It should be noted, that there are many different resources on the Internet, where the obtained data could be checked[41].

41 Example of the JavaScript for the Easter Sunday/Jewish Passover Calculator can be found here: https://www.staff.science.uu.nl/~gent0113/easter/easter_text2a.htm.

In the presented diagrams two values of the vernal equinox are given – March 20th and March 21st by the new style. As mentioned earlier, one of the criteria for the Easter date determination is the fixed date of the vernal equinox – it must be March 21st as on this exact date the real, astronomical vernal equinox was observed in IV AD when the I Ecumenical Council of Nicea took place. In reality, the date of the vernal equinox is changing with time. For example, during 2000 – 2054 years it is observed two times on March 21st, two times on March 19th and rest on March 20th (see. Table 6)[42]. However, in the examples mentioned above, the date of March 21st as the date of the vernal equinox, as required by the Church canonical rules, was strictly used.

Year	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Date (March)	20	20	20	21	20	20	20	21	20	20	20
Year	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
Date (March)	20	20	20	20	20	20	20	20	20	20	20
Year	2022	2023	2024	2025	2026	2027	2028	2029	2030	2031	2032
Date (March)	20	20	20	20	20	20	20	20	20	20	20
Year	2033	2034	2035	2036	2037	2038	2039	2040	2041	2042	2043
Date (March)	20	20	20	20	20	20	20	20	20	20	20
Year	2044	2045	2046	2047	2048	2049	2050	2051	2052	2053	2054
Date (March)	19	20	20	20	19	20	20	20	19	20	20

Table 6. Dates of the astronomical vernal equinox for 2000 – 2054 years.

Obtained results and their graphical representations confirm that the Alexandrian system due to its inaccuracies gradually moved behind the astronomical realities, to which the system was exactly intended to follow and which was achieved in the early history of the Church. On the other hand, the Gregorian Reformation greatly improved inaccuracies of the Alexandrian

42 The list of the spring vernal equinox dates and times for the period of 1788 - 2211 can be found here: http://ns1763.ca/equinox/vern1788-2211.html.

system and with an accuracy managed to return to the astronomical realities.

4 Conclusion

In the presented article the paschal question from historical, canonical, mathematical and astronomical points of view was considered. Namely, the historical development of the paschal formula, creation of the paschal tables, paschal/calendar systems' reforms and analyses of the comparative study of the Alexandrian and Gregorian paschalion with their corresponding astronomical data were presented. With this regard the following conclusions can be drawn:

- Development of the question concerning the determination of the Easter dates was historically a very long process. There were different practices in the early Church and one of them dictated direct dependence to the Jewish Passover. That was the reason for some Christian groups, which followed this tradition, to celebrate Easter twice in the same year. Exactly against this practice, the decisions of the I Ecumenical Council in Nicea were directed, and they stated full independence of the Easter date determination from the Jewish calculations. On the other hand, the intention was not to move the Easter celebration to the next Sunday in cases when the Easter Sunday coincided with the Jewish Passover. These coincidences happened many times in early Church until VIII century, but the Easter celebration was never moved to the next Sunday. Accidental coincidences did not matter at all, but what mattered was full independence from the Jewish calculations. Concerning the remark of "moving of the Easter celebration to the next Sunday", which can be found nowadays in some explanatory notes concerning the Easter date coinciding the Jewish Passover – it was a late addition to the question of the Easter date determination, which was based on misinterpretation of certain canonical rules of the Church and which were not

present in the canonical criteria for the Easter date determination.

- The Easter date determination is based on the astronomical phenomena. The purpose of the church calendar and system of the Paschal Tables was to provide the Church with the results being very close to the corresponding astronomical data of that time. These systems were considering the tropical year and the Moon phases. Due to the known fact that it is impossible to connect the tropical year and the Moon phases to each other precisely, any such system will feature certain inaccuracy. In this respect neither Julian calendar, nor 19-years Metonic cycle, or Paschalion/calendar system based on the Gregorian reform are exceptions. The main factor here is which calendar system gives less inaccuracy compared with its corresponding astronomical data based on the canonical criteria. Because the issue is directly and solely connected to the astronomical-mathematical apparatus, to check which system gives less inaccuracy does not represent a difficult task. Just to recall, after the lengthy disputes around the paschal question, the final winner was the Alexandrian school, which was the most advanced and leading scientific school at that time. Findings presented in this article proves that nowadays results given by the Gregorian paschalion are more precise than those provided by the Alexandrian paschalion. One of the main criticisms of the Gregorian paschalion from us, the Orthodox, is the fact that in some years the Easter Sunday calculated by the Gregorian method is previous to the Jewish Passover. However, historical evidence and different credible sources presented here prove that this case is not dealt with at all in the canonical criteria for the Easter date determination. Hence, this criticism is groundless.

- It was not a problem for the Church in the early centuries to adopt and use the system of the calendar and the Moon phases created and developed in the pagan world. It was not a problem for the Church to conduct the reformation in this field either. The calendar and the system of Moon phases do not represent

questions from the dogmatic field; they belong to the scientific field. The main purpose of such systems is to bring us closer to the early practice of the Church, which was using the real, astronomical phenomena of that time as the reference points for the Easter date determination.

Therefore, since it was not a problem for the Church to adopt and use the scientific method developed in the pagan world, why should it be an issue for us now to consider calendar reforms conducted in the Christian world, namely, Catholic world and to improve our own system in order to correct its inaccuracies and to not violate the Church's canonical requirements for the Easter date determination?

It should be noted here, that celebration of the Nativity feast was first established in the heretical circle, and only later it was spread and adopted by the Church and became its tradition[43]. The purpose of the Gregorian Reformation was to restore the early Church practice on the Paschal question. As it was already demonstrated here, by the Gregorian reform, it was possible to achieve a very high precision, while the accuracy of our Alexandrian method is decreasing with time. It is important to note, that the Gregorian reform did not produce any new paschal-calendar system, rather (than) it improved the existing Alexandrian system, which uses the Julian calendar and the 19-years Metonic cycle.

Moreover, it was mentioned here that there were few attempts to reform the current calendar in the Orthodox world too. The only outcome of these attempts was the replacement of the Julian calendar by the Revised Julian calendar, which in fact is more precise than the Gregorian calendar. However, even introducing the Revised Julian calendar was not quite successful in the Orthodox world – nowadays one part of the Orthodox world uses it, while another part follows original

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[43] Диакон Андрей Кураев, *Церковь в мире людей* (Издательство Сретенского монастыря, Москва, 2007), p. 299-301.

Julian calendar. However, in both cases, the Pachalion was untouched (except the Finnish Orthodox Church).

- In the beginning, the Gregorian reform met a quite negative response in the West. However, later it was accepted and gradually spread throughout Western Europe. It is known that one of the local Orthodox councils (in 1583) anathematized those who would follow the Gregorian reform[44]. However, this anathema does not prohibit a calendar reformation as such; it prohibits only following the Gregorian reform. On the other hand, this council was just a local council and not an Ecumenical Council. As previously noted, the Gregorian reform restored the criteria for the Easter date determination; it restored the principles used by the early Church in this question.

Hence, it does not sound logical to condemn such attempt by which restoring of old practice and tradition of the Church was possible. Moreover, the Gregorian reform was not dealing with the dogmatic question; its main purpose was to correct the Julian calendar and the 19-years Metonic cycle. Nowadays, the Finnish Orthodox Church (Jurisdiction of the Ecumenical Patriarchate) and at least one Western European parish (Jurisdiction of the Moscow Patriarchate) uses the new style for Paschal circle, hence, fulfilling the canonical requirements for the Easter date determination.

- It must also be noted that the attempt to justify and prove "correctness" of the calendar using certain miracles is completely wrong and leads to a logical dead-end. First of all, the calendar question does not belong to the dogmatic field of the Church. Second, the miracles happen where the Church is. Alexandrian Paschal Tables show that in some years the canonical requirements for the Easter date determination are broken. Hence, if one uses a certain miracle to prove "correctness" of the calendar, then with the same success one can show that the

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44 Протоиерей В. Ф. Хулап, Реформа календаря и пасхалии: история и современность (Церковный вестник, СПб, No. 3, 2002).

same miracle can "prove" possibility of breaking and not fulfilling the canonical requirements about the Paschal question. Here is the logical dead-end.

To conclude, the purpose of this article is to provide the reader with historical, canonical and scientific details and facts on the Easter date determination. This question is quite complex, and as Church history witness, it is a quite sensible subject within the Church – among the clergy as well as among parishioners. Unfortunately, quite often while discussing this topic one can hear wrong arguments or misinterpretations, which are justified neither scientifically nor historically. It is important to restore the correct understanding of this question, to realize its meaning and to follow the scientific analysis. Since this issue is not dogmatic by nature, it is possible to discuss and, if necessary, improve it according to the Church canons. There are many examples from the Church history when non-dogmatic questions were revised by the Church[45]. However, it is also important to bear in mind that any reform or attempt for the reformation in this direction must be made only by the unity of the Churches, by keeping *Sobornost*, to avoid any damage to the unity of the Church.

[45] Протопресвитер Николай Афанасьев, Неизменное и временное в церковных канонах (Живое Предание, Париж, 1937).